CANDIDATE'S NUMBER

(For the use of the Department only)

TM
60
c.2

Math 30X
1967

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HIGH SCHOOL AND UNIVERSITY MATRICULATION

EXAMINATIONS BOARD

DEPARTMENTAL EXAMINATIONS, 1967

MATHEMATICS 30X

Time — 3 hours.

DO NOT SPEND TOO MUCH TIME ON ANY ONE QUESTION.
READ THE QUESTIONS CAREFULLY.

Total Possible Mark — 161.

All work must be done in this booklet.
Do not write your name on this booklet.

Knott's Mathematical Tables supplied by the Presiding Examiner may be used. Candidates are NOT permitted to use slide rules.

Finished work in this booklet may be done in either pencil or blue or black ink (pen or ballpoint).

Do not put work which is to be read by the sub-examiner in the ROUGH WORK ONLY Column.

Solutions for all questions should be shown.
Some marks may be obtained for partial solutions.

DO NOT FOLD THE BOOKLET.

Place your test booklet in the answer envelope in such a way that the **UPPER LEFT HAND CORNER OF THE BOOKLET** (Candidate's Number) shows through the **SLASHED CORNER** of the answer envelope.

BE SURE TO SEAL THE ENVELOPE.

LB
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math.30X

CURR HIST

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Tests
D-2

Values

1. Solve for x: $x \in R$

1 (a) $(5 - x)(x + 2) \leqq 0$

1 (b) $|2x - 1| \leqq 5, \ x \in R$

1 **2.** Solve for x: $X \in {}^{+}R$

$\log_x 64 = 2$

1 **3.** Evaluate $-i^7, \ i = \sqrt{-1}$

1 **4.** Use image notation to describe

$g : x \rightarrow \sqrt{x} + 1, \ x \in R$

3 **5.** Write in standard form $a + bi, \ i = \sqrt{-1}$

$$\frac{(2 + i)(3 + 4i)}{4 + 2i}$$

FOR ROUGH WORK
(No marks given for work in this space)

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Values

FOR ROUGH WORK
(No marks given for work in this space)

3 **6.** Solve for n if $(n)_2 = \binom{n}{3}$, $n \in N$

2 **7.** Write in words, the meaning of

$$\sum_{j=-2}^{k} a_j$$

Values

3 **8.** If $b(0; 3, p) = b(1; 3, p)$ find the value of p.

FOR ROUGH WORK
(No marks given for work in this space)

2 **9.** Given events $A, B \in S^*$, what relation exists between A and B if $P(A \cap B) = P(B) \cdot P(A \mid B)$?

2 **10.** Convert 1.25 radian measure to degree measure.

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Values

FOR ROUGH WORK
(No marks given for work in this space)

11. If $f(x) = \frac{1}{x} + x^2$

1 (a) Evaluate $f(-2)$

2 (b) Evaluate $f(\frac{1}{x})$

4 **12.** Given that $Q \rightarrow P$ is a false statement, then

(a) the truth value of Q is ....

(b) the truth value of P is ....

(c) the truth value of (PVQ) is ....

(d) the truth value of $(PVQ) \rightarrow P$ is ....

2 **13.** If x, y and z are elements of R, *quantify* $x(y + z) = y + z$ so that the statement will be true.

Values

4 **14.** Let $A = \{ a, b, c \}$, and let the following table with respect to a binary operation # apply in A.

#	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Assuming the operation is associative, determine whether $A = \{ a, b, c \}$ and # form a group.

FOR ROUGH WORK
(No marks given for work in this space)

4 **15.** State the domain and range of $S = \{ (x, y) \mid y^2 = x^2 - 4,\ x, y \in R \}$

Values

3 **16.** If $f = \{(x, y) \mid y = 3^x, x \in R\}$, find f^{-1}

FOR ROUGH WORK
(No marks given for work in this space)

3 **17.** Determine the real value of k for which $kx^2 - 10x + k^2 - 15k = 0$ will have reciprocal roots.

Values

5 **18.** Find the equation in x with integral co-efficients which will have roots the squares of the roots of $-x^2 + 3x + 3 = 0$.

FOR ROUGH WORK
(No marks given for work in this space)

Values

5 **19.** A farmer has 50 acres of carrots. On July 15 he estimates that his crop would yield 200 bushels per acre, while the price is $5.00 per bushel. For every week he waits the crop increases by 10 bushels per acre while the price drops by 20¢ per bushel. When should he harvest his crop to gain maximum income if other factors are equal?

FOR ROUGH WORK
(No marks given for work in this space)

Values

FOR ROUGH WORK (No marks given for work in this space)

3 **20.** Use the identity $\cos^2 x + \sin^2 x = 1$ to prove $\tan^2 x + 1 = \sec^2 x$.

21. Given

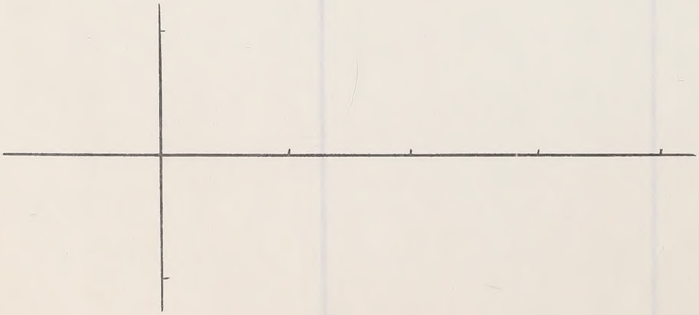
$$A = \left\{(\theta, y) \mid y = \frac{3}{4} \sin 2\theta,\ 0 \leq \theta < 2\pi,\ \theta, y \in R\right\}$$

3 (a) (i) the amplitude is ....

(ii) the period is ....

(iii) the phase shift is ....

2 (b) Sketch the graph of (a).



Values

5 **22.** Solve for x if $0 \leq x \leq \pi$, $x \in R$

$\sin 4x + \sin 2x = 0$

FOR ROUGH WORK
(No marks given for work in this space)

3 **23.** Simplify: $\dfrac{\sin A + \tan A}{1 + \sec A}$

Values

4 **24.** Use the formula for $\tan(x - y)$ to find the exact value of $\tan 15°$. (Leave your answer in simplest EXACT form.)

FOR ROUGH WORK
(No marks given for work in this space)

2 **25.** (a) Evaluate $|5 - 2i|$ $\quad i = \sqrt{-1}$

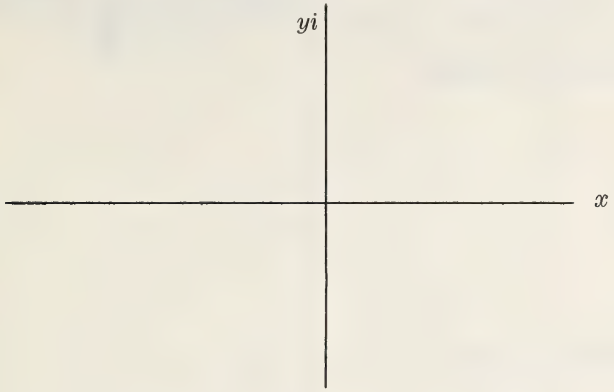
(b) If $z_1 = -i$ and $z_2 = 3 + 2i$ $\quad i = \sqrt{-1}$

2 (1) find $|z_1 - z_2|$

1 (2) What does $|z_1 - z_2|$ represent in the complex plane?

Values

3 **26.** Sketch the graph of $\{z \quad |z + i| = 2, z \in C\}$



4 **27.** Solve for x and y, $x, y \in R$, $i = \sqrt{-1}$

$$(3x - i)(1 + 2i) = (2 + i)(3 - yi)$$

FOR ROUGH WORK
(No marks given for work in this space)

Values

28. Given the relation

$A = \{(x, y) \mid x^2 + y^2 - 6x + 2y + 1 = 0, x, y \in R\}$

3 (a) Write the defining sentence in standard form.

1 (b) The graph of the relation is called

2 (c) The Domain of the relation is

2 (d) The Range of the relation is

3 **29.** (a) State the equations of the asymptotes of the graph defined by $4y^2 - 9x^2 = -36$.

3 (b) State the co-ordinates of the focal points of the graph in (a) above.

FOR ROUGH WORK
(No marks given for work in this space)

For ROUGH WORK
(No marks given for work in this space)

Values

4 **30.** Determine algebraically the set of ordered pairs for the following:

$$A = \{(x, y) \mid x - 2y - 3 = 0,\ 2x^2 + xy - y^2 = 0,\ x, y \in R\}$$

5 **31.** Do EITHER (a) OR (b).

(a) Given that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, if $A \cap B \neq \varnothing$, develop an expression for $n(A \cup B \cup C)$.

(b) Simplify: $\dfrac{n!}{(n-3)!\,3!} - \dfrac{(n-1)!}{(n-2)!\,1!}$

Values

4 **32.** (a) In how many ways may a play be cast from a class of 10 boys and 15 girls if the cast must contain 3 boys and 2 girls?

4 (b) If each student has the same chance of being chosen what is the probability that John and Mary, two of the students, will be chosen?

3 **33.** If six different letters are used to name the vertices of a regular hexagon, in how many ways may this be done?

FOR ROUGH WORK
(No marks given for work in this space)

Values

4 **34.** Prove: $\sum_{n=5}^{10} n = \sum_{i=1}^{6} (i + 4)$

6 **35.** Find the general term and the 17th term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{19}$

FOR ROUGH WORK
(No marks given for work in this space)

Values

6 **36.** Prove by mathematical induction that

$$\sum_{a=1}^{n} (a^2 + a) = \frac{n(n+1)(n+2)}{3}$$

FOR ROUGH WORK
(No marks given for work in this space)

Values

5 **37.** Use Knott's tables to find the probability a hunter has of hitting 4 ducks in flight with 6 shots if he downs 7 ducks with 10 shots on the average.

FOR ROUGH WORK
(No marks given for work in this space)

4 **38.** Independently, a coin is tossed, a card is drawn from a bridge deck and a die is thrown. What is the probability that we observe a tail on the coin, a king from the deck and a 3 on the die?

Values

4 **39.** In a certain school, examination results showed that 10% of the students failed English, 15% failed mathematics and 5% failed both mathematics and English. If there were 120 students writing the examinations, what is the probability that a student chosen at random failed English but not mathematics?

FOR ROUGH WORK
(No marks given for work in this space)

Values

7 **40.** A box contains 5 green and 8 blue light bulbs of the same size. If 4 light bulbs are drawn at random without replacement,

(a) find the probability distribution function of the number X of blue light bulbs drawn.

2 (b) find the most probable value of X?

FOR ROUGH WORK
(No marks given for work in this space)

Values

4 **41.** If X is the number thrown on a single uniform die, find $E(X)$ - - the mean value.

FOR ROUGH WORK
(No marks given for work in this space)

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DEPARTMENTAL EXAMINATIONS GRADE 12 1967 --

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Values

1 (1) If X is the [illegible] (SHOW WORK
uniformly [illegible] marks given for each
value. [illegible] in this space)